Linotype-Hell

Technical Information

Rational and Irrational

Within the past year there has been much discussion about halftone screening methods. Two terms that have been used extensively are *rational* and *irrational*. If you understand how these terms are used, you will be better able to understand halftoning methods in general. This is particularly important when you consider that three methods of halftoning are available in Linotype-Hell output devices: RT Screening*, HQS Screening*, and I.S. Technology*.

The factors that control screen angle and ruling cannot be discussed without some reference to trigonometry, so to start, let's look inside an imagesetter to see why trigonometry is so important.

If you wanted to plot out what happens when a laser imagesetter makes marks on film, you could do it very nicely on graph paper. At every intersection of two lines you have the ability to place a mark. The distance between the centers of two adjacent marks would be equal to the width of a grid square. (See Figure 1.)

The inverse of this width is called the addressability. Addressability describes the number of marks that an imageset-

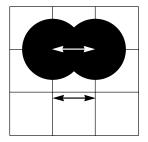


Figure 1 - Two marks on the grid of an imagesetter.

ter can make within a given area (usually an inch). For example, if the width of the grid square were 1/2540 of an inch, the addressability would be equal to 2540 dots per inch.¹ The important issue to remember is that the marks must be placed at regular intervals on the grid.

¹ The term addressability is often used interchangeably with the term resolution. For an in-depth discussion of the terms addressability and resolution, see the Linotype-Hell technical information piece called Addressability and Spot Size, part number 3071. Also discussed is the role of laser spot size.

Anything that is built on this grid must conform to the very nature of the grid. For example, to draw a line on the grid you must construct that line of points that fall on the grid.

If you look at Figure 2, you will see that the grid with the higher addressability is better able to produce a smooth line. Both lines show some amount of stepping, however as addressability increases, a better looking reproduction of the line is possible. Keep this concept in mind because as we will see, drawing a line on a grid is a very similar process to that of creating a digital halftone with a specific screen angle and ruling.

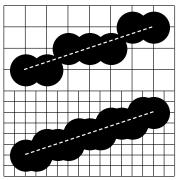


Figure 2 - Low addressability (top) and high addressability (bottom) grids. (For the purpose of this example, spot size remains the same at both addressabilities.)

The grid

Rational versus irrational

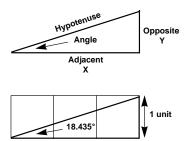


Figure 3 - Trigonometry terminology (top) and rational tangent angle of 18.435°, where the opposite side is one unit and the adjacent side is three units (bottom).

3 units

Digital halftones

Rational tangent

A rational number is one that can be defined as an integer² or a ratio of two integers. There is no in-between, the numbers must be integers. An angle with a rational tangent is one where (and here we get to the trigonometry part) the opposite and adjacent legs of the triangle are integers.

² Any positive or negative whole number, i.e., 1,2,3, or -1, -2, -3, as well as 0.

Imagesetter output is intrinsically rational because you can't build up anything on the grid unless you do it with marks that fit on the grid. If you think of each segment of the grid as an integer, you can see how this works. What then is irrational? In trigonometry it refers to the angles that can't be created with integers as sides of the triangle.

In Figure 3, an 18.435° angle can be easily created with a triangle that has an opposite side of one unit, and an adjacent side of 3 units. To get an angle of exactly 15° would require that the ratio of the opposite side to the adjacent

side equal .26794192. This number can't be recreated exactly with a simple ratio of integers. You can approximate a 15° angle with a ratio of integers, but the numbers need to get higher and higher for it to be very accurate. (See chart to right.)

Opposite	Adjacent		Resulting
Side	Side	Ratio	Angle
1 unit	3 units	.333333	18.435°
1 unit	4 units	.250000	14.036°
4 units	15 units	.266667	14.931°
11 units	41 units	.268293	15.018°
15 units	56 units	.267857	14.995°
(and so on…)			

A digital halftone cell is the imaginary boundary of a halftone dot.³ If you know some basic information about the halftone cell, you can use it to determine the screen angle and ruling of the halftone. The calculations can all be

done using basic trigonometry. For example, the screen angle can be calculated if you know the number of grid units that the halftone cell is tilted in the x and y directions. The screen ruling is equal to the inverse of the length of the side of the halftone cell.

Figure 4 shows a rational tangent digital halftone cell. Note how the corners of the digital halftone cell are fixed to the grid.

³ Refer to the Linotype-Hell technical information piece called Digital Halftone Dots, part number 3060, also see Measuring Screen Angle and Ruling, part number 3055.

In the simplest form of rational tangent screening every halftone cell is identical to its neighbor. Cells may be shifted to achieve a particular angle, or enlarged or reduced to make a different screen ruling, but each underlying cell will be identical. This screening method has simplicity on its side, but it has drawbacks. Because it is tied so strictly to the grid, not all angle and ruling combinations can be achieved. Some angles, 0° and 45° for example, are easy to achieve, but only at a limited number of screen rulings. (See chart to right.)

Other angles, particularly the 15° and 75° angles used in color separation, are much harder to achieve exactly. In the long run

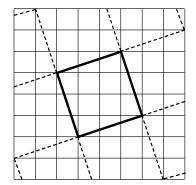


Figure 4 - Digital halftone cell.

Selected screen rulings where 0° and 45° angles occur with rational tangent screening at an addressability of 2540 dpi.				
Angle	Opp.	Adj.	Scr. Rul.	
0	0	21	120.95	
0	0	20	127.00	
0	0	19	133.68	
0	0	18	141.11	
0	0	17	149.41	
0	0	16	158.75	
45	15	15	119.74	
45	14	14	128.29	
45	13	13	138.16	
45	12	12	149.67	
45	11	11	163.28	

what this means is that it is impossible using this simple method to create four separations at 0°, 15°, 45°, and 75° where each separation has the same screen ruling.⁴

⁴ For more information on color separation, please refer to the Linotype-Hell technical information piece entitled Moiré, part number 3063.

To do color separation with rational tangent screening, a workaround is necessary. (This workaround was developed by engineers at Dr.-Ing. Rudolf Hell GmbH.) For the cyan and magenta separations, 18.435° and 71.565° selections are used instead of 15° and 75°. In addition, a 45° selection with a different screen ruling is used for the black separation, and a 0° selection with yet another screen ruling is used for the yellow separation. Although the screen rulings and angles are peculiar, this method works better than others because it is mathematically tuned (i.e., the screen angles and rulings are carefully chosen to reduce the amount of visible moiré.) Though this combination doesn't form a perfect rosette pattern, it gives results that are much better than trying to approximate conventional angles and rulings with rational tangent screening.⁵

⁵ The Linotype-Hell technical sample entitled What factors play a role in moiré?, part number 3064, illustrates this rosette pattern.

Dr.-Ing. Rudolf Hell GmbH (now Linotype-Hell) has played a key role in halftone screening technology. Many of these advances have been licensed to third parties. In fact, RT Screening was recently licensed to Adobe Systems for use in the PostScript** page description language.

One thing that was clear from early on with rational tangent screening was that if you made the halftone cell extremely large, you could get very close to a 15° or 75° angle. (Refer to the chart on the preceding page.) The problem was that the resulting halftone cell had an extremely low screen ruling. For example, at an addressability of 2540 if you attempted to get the 15.018° shown in the chart on the preceding page, the resulting screen ruling is 42.45 lines per inch. To get closer to 15°, the halftone cell must get even larger, and as a result, the screen rulings continue to drop. To effectively use halftone cells of this size, imagesetters would have to have addressabilities of 6,000 to 10,000 dots per inch.

A simpler solution is to do away with the requirement that every halftone cell fit precisely on the grid. If there were some way to make a huge halftone cell, and then subdivide it into smaller cells, you could get both an accurate angle, and a reasonable screen ruling. This is the concept behind the supercell. (See Figure 5.) Just exactly how a supercell is subdivided is of interest mainly to engineers and lawyers. Of more practical interest are issues related to quality, ease of use, and performance.

There are a number of manufacturers that use supercell rational tangent technology. The best way to judge their quality is to look at output. Linotype-Hell's supercell technology is called HQS Screening. Other methods include Adobe's Accurate Screens** and Agfa's Balanced Screening.

For ease of use and performance, a screen cache is essential. Because of the large size of the supercell it is important to devise a method to speed the calculations necessary to create halftone dots. Linotype-Hell deals with this issue in its PostScript RIPs (Raster Image Processors) by using a screen cache. A screen cache works like a font cache. Once a screen has been calculated, it is stored in the screen cache and recalled when needed.

The use of a screen cache means that user's are not limited to a menu of pre-calculated screen sets (with predetermined spot functions.) Pre-calculated screen sets take up space on the RIP even if you never use them.

RT Screening

Supercell rational tangent

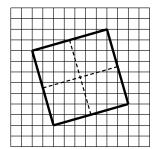


Figure 5 - A supercell subdivided into smaller cells. Notice how the corners of the supercell fit on the grid while not all the corners of the smaller cells do. In addition, the number of spots that fall within each of the smaller cells may vary from cell to cell.

The screen cache

Irrational screening	Even though a supercell achieves angles and rulings much more accurately than a simple cell, it still is based on cells that repeat. If a screening technolo- gy is truly irrational, there would be no cell (or supercell) and each halftone dot would be calculated separately rather than being part of a system of cells or supercells. This lack of a cell or supercell means that huge numbers of cal- culations need to be done on the fly. Handling this huge amount of calcula- tions is one of the requisites of a truly irrational screening system. In addition, because an irrationally screened halftone is being created on a grid, there are times when rounding-off errors cause artifacts. To avoid these artifacts, some sophisticated optimization techniques must be used.
	To summarize:
	 With rational tangent methods, the grid is paramount. Therefore, repeating cells or supercells must be found that fit on the grid.
	 With irrational tangent methods the screen ruling and angle are paramount. Everything possible is done to assure that they can be achieved, despite the limitations of the grid.
	Devices that employ irrational screening include the Linotype-Hell DC 3000 product family and the newly introduced Linotype-Hell RIP 60 with I.S. Technology. These devices are capable of using both rational tangent and irrational tangent screening. This gives users the ability to choose the screening method that fits their requirements.
Device dependence	Throughout this discussion it should have become clear that halftone screen- ing is device dependent. In other words, the addressability of the device plays a significant role in the screen angles and rulings that can be achieved. This is why any type of screening should be tuned to a given output device.
Uses for screening methods	Screening methods are often more equipment dependent than application dependent, however, here are some guidelines for the different methods:
	 RT Screening (i.e., rational) - Single color work, duotones, tritones, cartoon color (i.e., 3-colors plus black line work), and process color work where the aforementioned rosette pattern is acceptable.
	 HQS Screening (i.e., supercell rational) - All of the above plus higher quali- ty process color work.
	 I.S. Technology (i.e., irrational) - All of the above plus the highest quality color work.
Conclusion	As a final thought, be careful when you compare different types of screening. For example, the content of an image may be either easy or difficult to repro- duce without moiré. The quality of the scan also plays a role. To make a fair comparison of screening methods, samples should be prepared from the same scan data, at comparable screen rulings, and if at all possible at com- parable imagesetter addressabilities.
Comments	Please direct any questions or comments to:
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